Stochastic Power/Ground Supply Voltage Analysis via Moment and Correlation Computation by Statistical Transient Toggling Analysis

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Outline

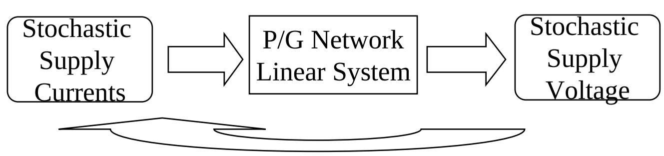
- Background
- Problem Formulation
- Stochastic Power/Ground Network Analysis
- Statistical Toggling Analysis
- Statistical Transient Toggling Analysis (STTA)
- Experiments
- Conclusion

P/G Supply Voltage Integrity Analysis

- Increasing Power/Ground supply voltage degradation in latest technologies
  - IR drop (DC/AC)
  - L dI/dt drop
- Effects:
  - Malfunction
  - Performance degradation
- P/G supply networks are special interconnects
  - Complex topology, numerous nodes, IOs
- Scalability improvement schemes
  - Top-down: multigrid-like, hierarchical, partition
  - Bottom-up: random walk

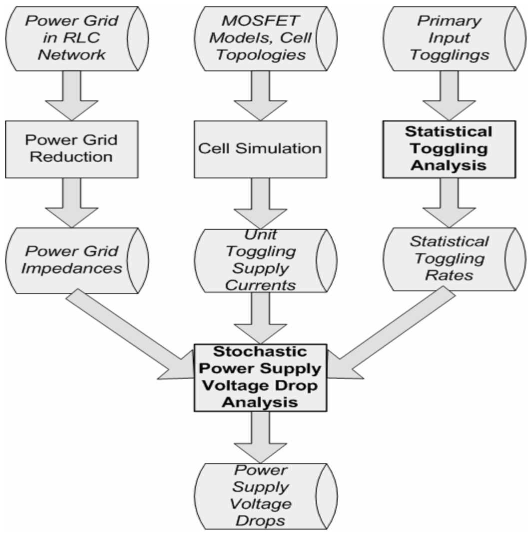
Stochastic P/G Analysis

- An RLC P/G network is a linear system
- Statistics of supply currents
  - Performance variation due to inputs, PVT, …



- Worst case analysis results have little occurrence probability
- In practice, a *scaling factor* is applied
- Stochastic P/G analysis by Blauuw et al.

Stochastic P/G Analysis Flow



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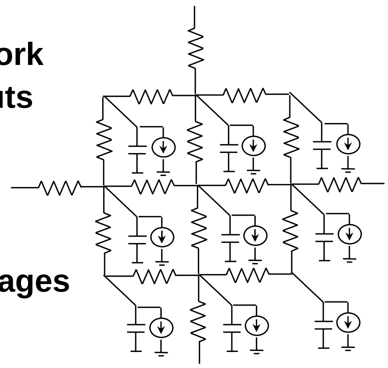
Stochastic P/G Analysis

- Given
  - an RLC P/G supply network
  - statistics of primary inputs
  - transistor-level netlist
  - transistor models
- Find
  - stochastic P/G node voltages



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Stochastic P/G Network Analysis

- **Pant, Blauuw, Zolotov, Sundareswaren, and Panda, ``A Stochastic Approach to Power Grid Analysis,'' DAC'04.**

$$V_p = \sum_j \int_0^\infty I_j(t-\tau) h_{pj}(\tau) d\tau$$

$$E(V_p) = \sum_j \int_0^\infty E(I_j(t-\tau)) h_{pj}(\tau) d\tau = \sum_j E(I_j) \int_0^\infty h_{pj}(\tau) d\tau$$

$$E(V_p^{\,2}) = \sum_j \sum_k \int_0^\infty \int_0^\infty R_{jk}(\tau_1 - \tau_2) h_{pj}(\tau_1) h_{pk}(\tau_2) d\tau_1 d\tau_2$$

$$R_{jk}(\tau_1 - \tau_2) = E(I_j(\tau_1) I_k(\tau_2)) = \mathrm{cov}(I_j(\tau_1), I_k(\tau_2)) + \mu(I_j(\tau_1))\mu(I_k(\tau_2))$$

- ***How to obtain supply current statistics?***

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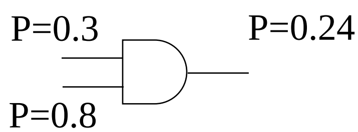
Signal Probability

- **P(*j*) = Probability for a signal *j* to be of logic one**
- **P(*a&b*) = P(*a*) & P(*b*)**
- **P(*a|b*) = P(*a*) | P(*b*)**

$$P(y) = P(x_i)P(y|_{xi=1}) + (1 - P(x_i))P(y|_{xi=0})$$

- **Linear time computation of BDD size**

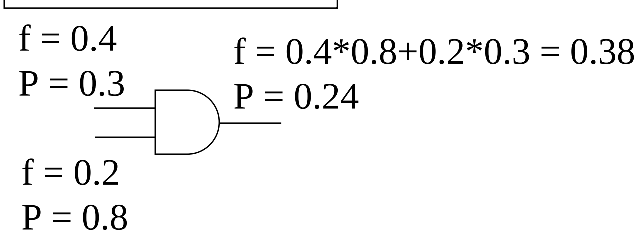


Signal Toggling Rate

- ***f_j* = statistical variable for the per unit time signal toggling number for a gate *j***

$$f_y = \sum_i P\left(\frac{\partial y}{\partial x_i}\right) f x_i$$

$$\frac{\partial y}{\partial x_i} = y|_{xi=1} \oplus y|_{xi=0}$$



Moments and Covariance

- **Linear time computation in a single netlist traversal**

$$\mu(f_y) = \sum_i P(\frac{\partial y}{\partial x_i})\mu(f_{x_i})$$

$$\sigma^2(f_y) = \sum_i P^2(\frac{\partial y}{\partial x_i})\sigma^2(f_{xi}) + 2\sum_{i,j} P(\frac{\partial y}{\partial x_i})P(\frac{\partial y}{\partial x_j})\operatorname{cov}(f_{xi}, f_{xj})$$

$$\operatorname{cov}(f_y, f_z) = \sum_i P(\frac{\partial y}{\partial x_i})\operatorname{cov}(f_{xi}, f_z)$$

Stochastic Transient Toggling

- ***$f_y(t)$* = statistical toggling rate for gate *y* at time *t***

$$\mu(f_y(t)) = \sum_i P(\frac{\partial y}{\partial x_i})\mu(f_{x_i}(t-d_g))$$

$$\sigma^2(f_y(t)) = \sum_i P^2(\frac{\partial y}{\partial x_i})\sigma^2(f_{xi}(t-d_g))$$

$$+2\sum_{i,j} P(\frac{\partial y}{\partial x_i})P(\frac{\partial y}{\partial x_j})\operatorname{cov}(f_{xi}(t-d_g), f_{xj}(t-d_g))$$

$$\operatorname{cov}(f_y(t_1), f_z(t_2)) = \sum_i P(\frac{\partial y}{\partial x_i})\operatorname{cov}(f_{xi}(t_1-d_g), f_z(t_2))$$

- ***d_g* = gate delay**
- ***$f_y(t)$* can be represented in polynomials for efficiency**

Stochastic Supply Current

- **Convolution of transient toggling rate and unit (per toggling) supply current gives stochastic supply current**

$$I = \int_0^\infty I_{unit}(t-\tau) f_y(\tau) d\tau$$

Algorithm

Input: input signal probabilities, input toggling statistics

Output: Supply voltage moments

1. Traverse the netlist, for each gate *j*
2. Compute signal probability P(j)
3. Compute μ, σ, cov of togging
4. Compute stochastic supply current I_j
5. Compute supply voltage drop moments

Outline

- **Background**
- **Problem Formulation**
- **Random Walk**
- **Moment Computation in an RLC Tree**
- **SMM Theory**
- **Experiments**
- **Conclusion**

Experiments

- ISCAS'85 benchmark circuits
- Power mesh of R=0.2Ω, Ip=0.05mA in 90nm technology
- 5-value logic simulator taking glitches into account

I. $P = 0.5, \mu(f) = 0.1, \sigma(f) = 0.09$

II. $P = 0.5, \mu(f) = 0.1, \sigma(f) = 0.09$

	STTA			*10,000x Monte Carlo*			*100x Monte Carlo*		
(I)	*Vdr*	*σ(Vdr)*	*CPU*	*Vdr*	*σ(Vdr)*	*CPU*	*Vdr*	*σ(Vdr)*	*CPU*
s1196	*127.51*	*102.02*	*2.57*	*113.01*	*97.98*	*103.00*	*97.02*	*91.15*	*3.25*
s1238	*120.07*	*96.78*	*2.47*	*108.59*	*93.01*	*95.59*	*102.47*	*97.04*	*3.29*
(II)	*Vdr*	*σ(Vdr)*	*CPU*	*Vdr*	*σ(Vdr)*	*CPU*	*Vdr*	*σ(Vdr)*	*CPU*
s1196	*600.36*	*161.25*	*2.57*	*538.90*	*152.67*	*103.00*	*512.44*	*133.65*	*3.25*
s1238	*637.54*	*169.99*	*2.47*	*564.82*	*160.94*	*95.59*	*540.41*	*141.38*	*3.29*

Summary

- *Stochastic everything*
  - *P/G supply voltage*
  - *Timing*
  - *Power*
- *A complete stochastic P/G supply voltage analysis flow*
- *Statistical Transient Toggling Analysis (STTA):* an extension of *signal toggling analysis in power estimation* for supply current statistics
- Statistical methods are much more efficient than Monte Carlo simulation

Thank you !